Exercises: Bayesian Inference for Discrete Random Variables

Exam Relevant

Question 1: Let Y be a discrete random variable with probability function given in the following table

y_i	$f(y_i)$
1	0.1
2	0.2
3	0.1
4	0.2
5	0.4

1. Calculate $P(1 < Y < 5)$
2. Calculate $E(Y)$ and $Var(Y)$
3. Suppose $W = 2 * Y + 5$. Calculate $E(W)$ and $Var(W)$
4. Calculate $E(Y + W)$ and $Var(Y + W)$

Question 2: There is a box containing a total of 4 balls, some of which may be red and the rest of which are green. You do not know how many of the balls are red, so assume that your prior probability is equal across possible values of X.

1. You draw a ball randomly from the box and the selected ball is red. Calculate the posterior distribution probability. Also Prove that:
   - ◇ Multiplying all the prior probabilities by a constant does not change the result of Bayes' theorem.
   - ◇ Multiplying the likelihood by a constant does not change the result of Bayes' theorem.

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2. **Without replacement:** You take the second ball from the box (without replacing the first ball) and it is red again. Calculate the posterior distribution probability in two ways: "Analyzing the observations sequentially one at a time" and "Analyzing the observations all together in a single step". Prove that they produce the same result.

3. **With replacement:** You take the second ball from the box (after putting the first ball back to the box) and it is red again. Calculate the posterior distribution probability in two ways: "Analyzing the observations sequentially one at a time" and "Analyzing the observations all together in a single step". Prove that they produce the same result.

Question 3: Let $Y \sim Poisson(\mu)$. Suppose that we believe there are only four possible values for μ, 1,1.5,2, and 2.5. Suppose we consider that the two values, 1 and 2, are twice as likely as the two values 1.5 and 2.5. Suppose $Y = 1$ was observed. Calculate the posterior distribution probability.

Question 4: Suppose your evil twin brother has two coins: one comes up heads 30% of the time and the other 50% of the time. He then comes to you with some coin, you're not sure which one and he wants to make a bet with you. Betting money that if it comes up heads, you win. You're not sure of it's the loaded coin or if it's just a fair one. You set your prior probability about each coin: 0.6 for the loaded coin and 0.4 for the fair coin. Your brother gives you a chance to flip it three times and just check it out. You get two heads and one tail. Which coin do you think it is and how sure are you about that?

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Bonus Coding

Exercises — Data visualization (from R for Data Science)
See `https://r4ds.hadley.nz/data-visualize.html` for more details.

Exercises

1. How many rows are in `penguins`? How many columns?

2. What does the `bill_depth_mm` variable in the `penguins` data frame describe? (Read the help page with `?penguins` to find out.)

3. Make a scatterplot of `bill_depth_mm` vs. `bill_length_mm`. That is, make a scatterplot with `bill_depth_mm` on the y-axis and `bill_length_mm` on the x-axis. Describe the relationship between these two variables.

4. What happens if you make a scatterplot of `species` vs. `bill_depth_mm`? What might be a better choice of geom?

5. Why does the following give an error and how would you fix it?

```
ggplot(data = penguins) +
  geom_point()
```

For additional exercises and examples on data visualization with `ggplot2`, see the online chapter at `https://r4ds.hadley.nz/data-visualize.html`.